Enhancing selectivity of plate-type electrostatic separators using non-dominated sorting genetic algorithms (NSGA-II)

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Abstract: The paper presents a novel application of non-dominated sorting genetic algorithms (NSGA-II) to optimise the performance of plate-type electrostatic separator; an environmental friendly technique for selective sorting of conductive from nonconductive constituents of a granular mixture. As several decision variables control detachment of the particles from the plate; hence the separator's selectivity, NSGA-II is applied to determine their optimal values subject to simultaneous satisfaction of two proposed objective functions. These functions aim to maximise the separation distances, while maintaining the detachment fields, for different species. A GA-optimised charge simulation algorithm was developed to enable computations of detachment fields and positions of the particles. Two extreme solutions encompassing the other Pareto results are examined and analysed. The study illustrates the applicability of NSGA-II in solving the complex multiobjective optimisation problem of electrostatic separators in order to facilitate new development and designs of this environmental friendly technology.

Keywords: evolutionary computations; multiple objective programming; non-dominated sorting genetic algorithms; NSGA-II; electrostatic separation processes; innovative computing and applications.

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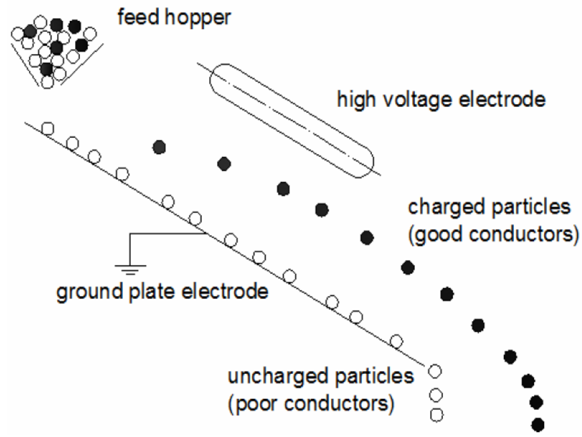
1 Introduction

Electrostatic separation has been extensively researched and developed over the last two decades as an important class of material-processing technologies; mainly used for the selective sorting of solid species by means of the electric forces which act on charged or polarised bodies (Iuga et al., 2011). Often, the various designs were evolved on the basis of numerous experimental investigations, practical experience and insight (Das et al., 2007; Medles et al., 2007; Vlad et al., 2000). Although these approaches have worked well in certain circumstances, they may not always lead to the best possible designs. Less efforts, however, have been directed towards quantitative application of the principles of engineering computational techniques. Now, increasing demands and competition require the use of suitable mathematical models describing the operation of electrostatic separators, as well as their use, with modern tools of optimisation to give the best designs. In this work, we present a methodology for obtaining the best possible designs for electrostatic separator through optimisation of multiple objective functions simultaneously. A recently developed AI-based computational technique, non-dominated sorting genetic algorithms (NSGA-II) (Deb, 2011a; 2011b), coupled with a suitable mathematical model (Vlad et al., 1998, 1999), is employed to achieve this goal. The proposed approach would be of significant assistance for experimental and laboratory investigations and designs of electrostatic separators and would facilitate the feasibility studies leading to the development of new applications of this technology. It is believed, to the best of our knowledge, that such a multiobjective optimisation study on electrostatic separators has not been reported earlier in literature.

In plate type electrostatic separators, the electric field is generated between a tubular electrode connected to a dc high-voltage supply and a grounded electrode (Figure 1). When the granular material to be separated is fed onto the surface of the plate, the insulating particles slide along the plate without being affected and are recovered in the left part of a collector box. The conducting particles, charged by electrostatic induction in contact with the grounded plate, are attracted by the high-voltage electrode and are recovered in the right part of the collector box. The particle trajectories, and hence the quantity and purity of the separated products, can be controlled through a compromise of several design parameters including: the slope and length of the high-voltage electrode, the gap distance and the applied voltage (Dahou et al., 2011; Vlad et al., 1998, 1999, 2003). These separators are suitable for removing the metallic particles that impurify the acrylonitrile butadiene styrene (ABS) products recycled from information technology wastes (Iuga et al., 2011). Optimum and enhanced performance can, only, be achieved by a favourable overall design of the whole set of the performance parameters (Vlad et al., 2000) which is the purpose of this work.

Figure 1 Schematic representation of a plate type electrostatic separator



While simple genetic algorithms (SGAs) are suitable for optimisation problem involving a single objective function nevertheless, for problems involving multiple objective functions, there may exist a set of several equally desirable optimal points (Goldberg 1989; Sivanandam and Deepa, 2008). These solutions are referred to as Pareto sets or non-dominated solutions; and anyone of them could be selected for design or operation. These Pareto sets are of significant importance in narrowing down the choices to be considered by designers. One of the most advanced and robust techniques suitable for solving such multiobjective optimisation problems is the NSGA-II (Deb et al., 2002; Haupt and Werner, 2007; Sahoo et al., 2009; Subramanian et al., 2009). The CSM; on the other hand, is a numerical method for electric field computation and SGA is, here, used for determining the optimal locations of the simulation charges of the charge simulation method (CSM) and their values through a developed computer code referred to here as (SGACSM) code(Abouelsaad et al., 2009, 2011).

In order to optimise an industrial separation process, the operating conditions should simultaneously satisfy several criteria while taking into consideration the fact that the process performance depends on a large number of variables (Dascalescu et al., 2004). Two realistic objective functions are proposed to achieve this purpose. The first, F_1, aims at modulating the field profile along the ground plate. The second objective function F_2, meanwhile, is to maximise the separation distance at which particles will detach from the plate; thus allowing better selectivity in the separator performance**.** A simplified mathematical model expressing the correlation between the process (e.g., detachment field, detachment position and separation distance) and the related decision variables is employed in the analysis (Das et al., 2007; Iuga et al., 2011; Medles et al., 2007; Vlad et al., 2000). The present study aims to demonstrate and analyse, for the first time, the applicability of NSGA-II in the case the complex multiobjective optimisation problem of plate type electrostatic separators. The trial solutions are classified into several fronts on the basis of the concept of non-dominance with assigned appropriate fitness values. The techniques of NSGA-II are then used to obtain the

Pareto optimal set. A short summary of NSGA-II distinct features is provided within the paper.

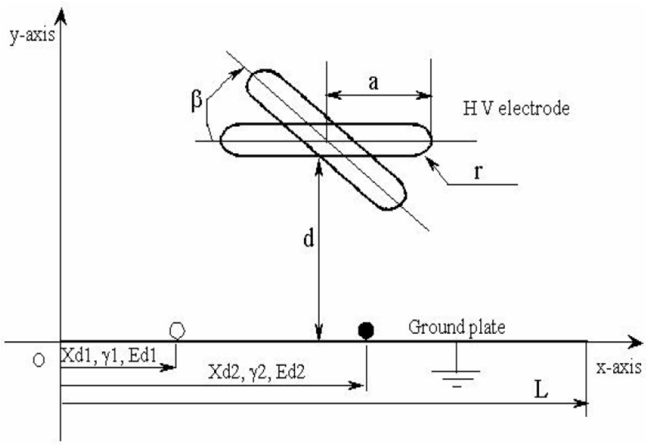
2 Method of analysis

The performance of the electrostatic separation is correlated to the capability of controlling the detachment of the particles from the plate electrode. The amount of charge carried by the particles and the force which acts upon them are determined by the strength of the electric field in the active zone of separator. A simplified mathematical model of the electrostatic separation, devised in Vlad et al. (2000), is here utilised.

The electrode arrangement with particles of mass densities γ_1 and γ_2 located on a ground plate of length L is shown in Figure 2. The gap distance is d, the slope of the high-voltage electrode is β, the half length of the high voltage electrode is a. SGA-CSM computer code is developed in order to compute the electric field strength E_{com} in a series of points along the surface of the plate electrode, for an applied base voltage U_{com}. The distance between any two adjacent points i and j was taken 1mm. The electric field strength E for a different applied voltage, U, can be evaluated by using the scale factor $k_{com} = U / U_{com}$ as;

$$E = k_{com}\ E_{com} \tag{1}$$

Figure 2 Representation of the electrode system of a plate type separator



A particle detaches from the plate electrode at the point where

$$E = E_d; \tag{2}$$

or,

$$E_{com} = E_d / k_{com} \tag{3}$$

where E_d is the detachment field strength. The following assumptions are usually employed in literature to simplify the analysis (Vlad et al., 2003):

1. the particle is a perfect conducting sphere
2. the particle charges instantly through electrostatic induction
3. the distance between two adjacent particles is large and each particle can be approximated by a material point
4. the space is homogeneous, isotropic with the permitivity ε
5. the electric force before the detachment is $F = 17.2\ \varepsilon\ r^2 E^2$ (Felici, 1966).

The detachment field is then expressed as:

$$E_d = 0.493[(r\gamma\ g / \varepsilon)\ \cos\alpha]^{1/2} \tag{4}$$

where ε being the permittivity of air, r being radius of particle, γ is the mass density, α is the inclination angle of the ground plate usually fixed at 60° for practical type separators and g = 9.81 m s^{-2}. For the case of a horizontal ground plate, α = 0, as shown in Figure 2 the detachment field becomes;

$$E_d = 0.493[(r\gamma\ g / \varepsilon)]^{1/2} \tag{5}$$

Let E_{comi} and E_{comj} be, respectively, the electric field strength at two successive points of coordinates X_i and X_j on the plate electrode. Then, if $E_{comi} < E_d / k_{com} < E_{comj}$, the position X_d of the detachment point can be estimated by linear interpolation as

$$X_d = X_i + \left(X_j - X_i\right) \left[\left(\left(E_d / k_{com}\right) - E_{comi}\right) \middle/ \left(E_{comj} - E_{comi}\right)\right] \tag{6}$$

The SGA-CSM code is used to compute these fields as well as the detachment field and detachment position of the different particles. The code furnishes the required data for the multi objective algorithm described next.

The NSGA-II programme is used for optimisation of two proposed objectives functions aiming to enhance the performance of the separator. Assuming that; for the same radius of particle and different mass densities γ_1, γ_2, the detachment fields are E_{d1}, E_{d2}, and detachment distances are X_{d1} and X_{d2}, then, two objective functions are constructed as follows: The first, F_1, aims at modulating the field profile along the ground plate in such a way as to establish an average field E_{av1} equal to E_{d1} on the first half of the plate and an average field E_{av2} equal to E_{d2} on the second half. The second objective function F_2, meanwhile, is to maximise the separation distance $(X_{d2} - X_{d1})$; thus allowing more selectivity in the separator performance. As the codes for SGA and NSGA-II usually work with minimisation of objective functions and as one of our two objective functions involves maximisation, the problem is converted into a pure minimisation problem by defining fitness functions, F_1 and F_2, both of which are to be minimised. The overall goal is to enhance the selectivity; hence the efficiency and performance, of the separator through optimisation of the operating parameters by minimising the following two objectives functions:

$$\text{Min } F_1 = w_1 f_1 + w_2 f_2; \tag{7}$$

where

$$f_1 = \left(\text{abs } \left(E_{av1} - E_{d1}\right)\right) / E_{av1}; \tag{8}$$

$$f_2 = \left(\text{abs } \left(E_{av2} - E_{d2}\right)\right) / E_{av2}; \tag{9}$$

and

$$\text{Min } F_2 = 1 / \left(\text{abs } \left(X_{d1} - X_{d2}\right)\right); \tag{10}$$

The weighting functions w_1, w_2 are taken here to be both equal to 0.5. The actual granular mixtures are rarely composed of particles of same size or same mass density. In general, the radii r of the particles of the mixtures are comprised between two limits: $r_{min} < r < r_{max}$ with mass density between γ_1 and γ_2. As a general rule, a compromise has to be found between the gap distance d, the slope of the high-voltage electrode β, the length of the high voltage electrode a, and the applied voltage U, in order to facilitate the separation process of mixtures. The optimum set depends on the characteristics of each particulate mixture that is being processed in a plate type separator.

3 Algorithm for electric field computation

The 2-D computational domain for plate type electrostatic separator is shown in Figure 2. The electric field of a plate type separator is described by the Laplace equation

$$\nabla^2 V = 0 \tag{11}$$

The CSM (Malik, 1989; Singer et al., 1974) is an effective numerical method for calculating the electric field intensity of open-boundary problem (Elrahman, 2011; Gururaj and Kishore, 2008). The CSM computes the simulating charge magnitudes by satisfying the boundary conditions at a selected number of contour points along the electrodes surfaces. The unknown charges are computed by setting up the simultaneous equations

$$[P]\,[Q] = [V] \tag{12}$$

$$\left[F_X\right][Q] = \left[E_X\right] \tag{13}$$

$$\left[F_Y\right][Q] = \left[E_Y\right] \tag{14}$$

where [P] is the potential coefficient matrix, [Q] is the column vector of unknown charges; [V] is the column vector of known potentials at the contour points. $[F_X]$ and $[F_Y]$ are the electric field intensity coefficients between the simulation charges and the electric field intensity components at the point where the electric field intensity is required in a Cartesian coordinate system. $[E_X]$ and $[E_Y]$ are the components of the electric field intensity at the same point. In the present calculations, N_l number of line charges are used to simulate the high voltage electrode and the image of these charges are used to simulate the ground plate. An algorithm is developed to determine an appropriate arrangement of these fictitious simulating charges. Figure 3 shows the arrangement of the simulating charges inside the high voltage electrode and their images. Three optimisation parameters were assumed for optimal allocation of the simulating charges as shown in Figure 4. The role of SGA is to determine the values of these parameters so as to achieve a desired accumulated square error (ASE) in the potential values on the surface of high voltage electrode. The ASE value is obtained by evaluating errors at N_h check points uniformly distributed on the electrode surface. The objective function, FF, in this case has the form

$$FF = \sum_{j=1}^{N_h} \left[U - u_j \right]^2 \tag{15}$$

where U is the electrode voltage, u_j is the potential obtained by the CSM and N_h is the total number of check points. The bit sequences of four individuals of the '0th generation' and the initial condition, are created by using uniformly random numbers. The calculations are terminated when a prespecified number of generations is reached. The sequence of the algorithm is as follows:

1 determine the domain for the optimisation parameters
2 SGA generates initial uniform random values for C_1, C_2, C_3
3 for each call to the CSM routine by SGA, the CSM will produce the ASE error for these optimisation parameters
4 the SGA will then evaluate the fitness function and modify the optimisation parameters to minimise the ASE error
5 Steps 3 and 4 are repeated for a prespecified number of generations.

Figure 3 The CSM representation of the electrode system of a plate type electrostatic separator

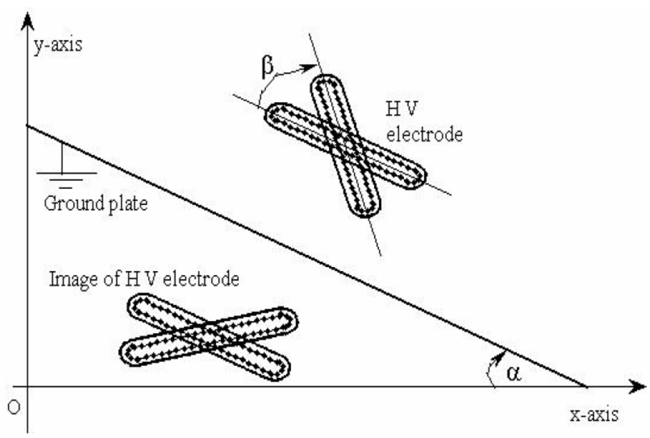
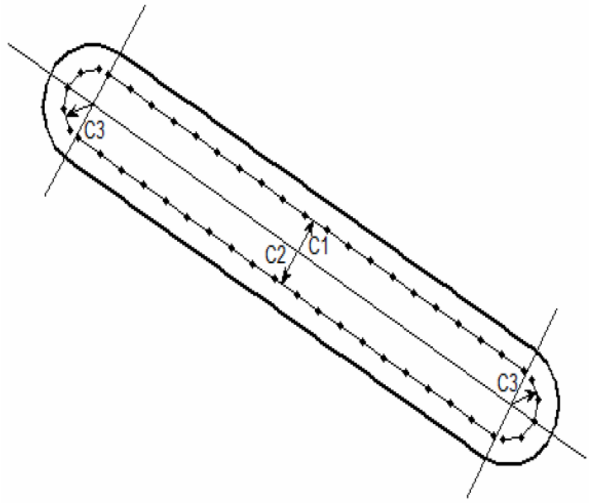


Figure 4 The optimisation parameters for the arrangement of the simulating charges in the CSM



An example for field computations using the SGA-CSM algorithm is demonstrated in this section. A model plate-type separator with the dimensions L = 0.380 m, a = 0.075 m, d = 0.075 m, r = 0.013 m, α = 60°, β = 30° is simulated. The bounds for the CSM optimisation parameters, C_1, C_2, C_3, are taken between 0.1 r and 0.9 r. The total number of simulating charges, N_s, the total number of contour points, N_c, and the total number of check points, N_h, are 50, 50, 100 respectively. The data of SGA-CSM, is given in Table 1. SGA is utilised to find the optimal values of C_1, C_2, C_3 so as to achieve an ASE of less than 10^{-5}. An example of the optimal values of the three parameters C_1, C_2, C_3 for d = 0.080 m, α = 60°, β = 30°, U = 10 kv are 0.4485, 0.4993, 0.5487 r. Also the optimal values of the three parameters C_1, C_2, C_3 for d = 0.065 m, α = 60°, β = 0°, U = 10 kv are 0.4958, 0.4966, 0.5130 r. The variation of the electric field strength at the surface of the plate for different values of d, Figure 5, and for different values of β, Figure 6, is computed. These distributions are in good agreement with those obtained earlier using the boundary element method (Vlad et al., 2000) which, further, confirms the accuracy of the developed SGA-CSM algorithm. A plot of the equipotential contours, for the case of β = 30°, is also shown in Figure 7. The developed SGA-CSM algorithm could; thus, serve as a useful and efficient tool for field computation of electrostatic separators.

Table 1 The SGA-CSM parameters

Number of simulating charge of HV electrode	50
Number of optimisation variables, C_1, C_2, C_3	3
Number of bits per variable per charge	14
Population size	4
Mutation rate	0.2
Number of generation	100
Using roulette wheel selection and single point crossover	

Figure 5 Variation of the electric field strength at the surface of the plate, for α = 60°, β = 30°, U = 10 kv, and various gaps d (see online version for colours)

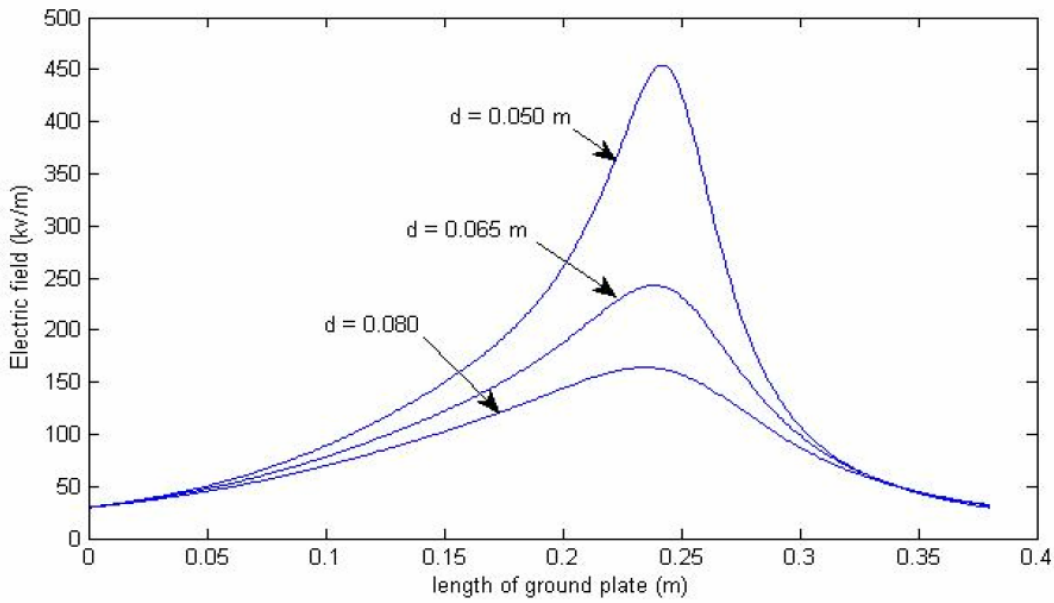


Figure 6 Variation of the electric field strength at the surface of the plate, for α = 60°, d = 0.065 m, U = 10 kv, and various β (see online version for colours)

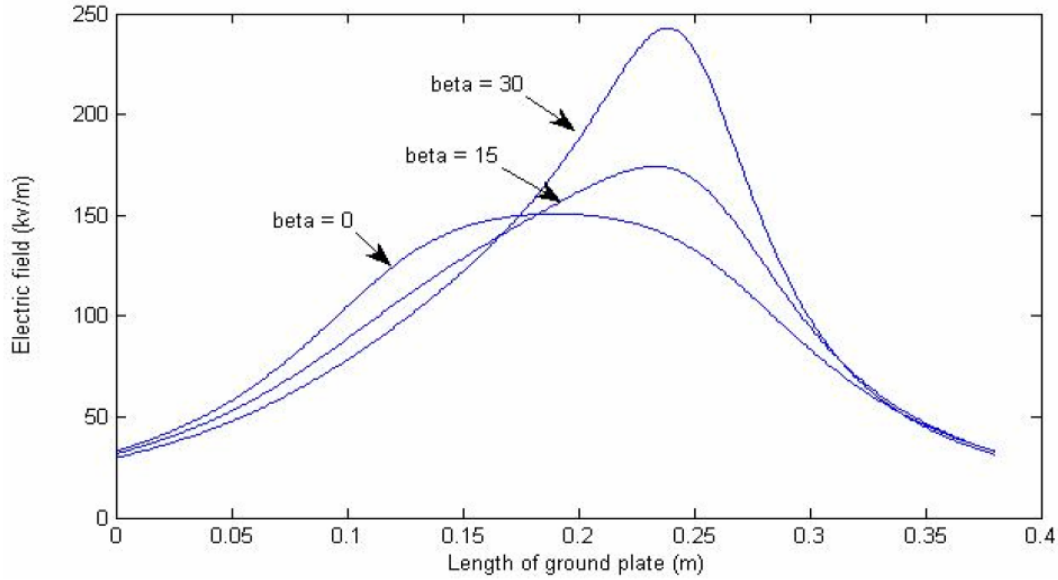
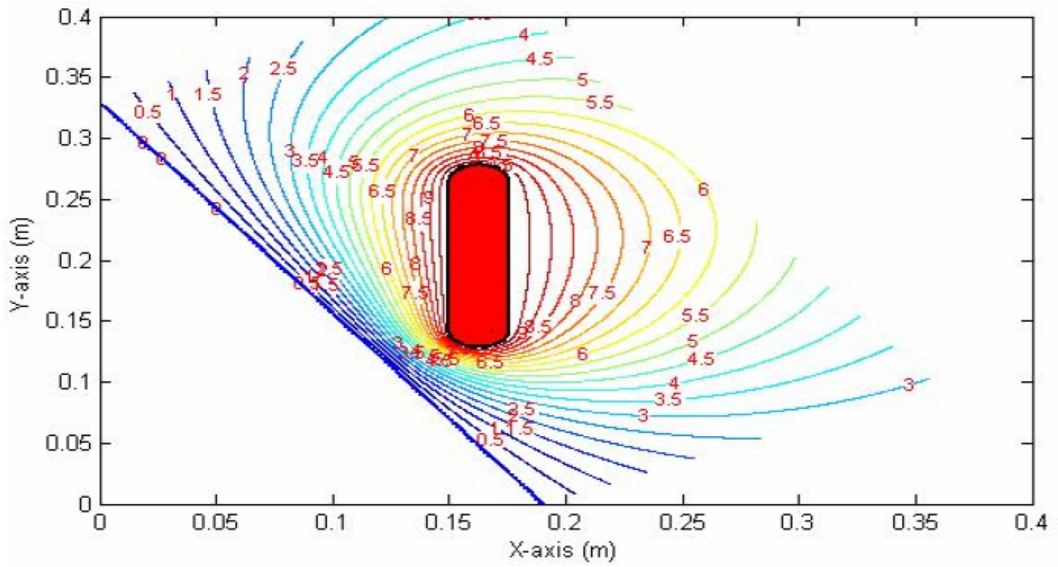


Figure 7 Equipotential lines of the electric field in the basic electrode arrangement for α = 60°, β = 30°, d = 0.065 m and U=10 kv (see online version for colours)



4 Non-dominated sorting genetic algorithms

Kalyanmoy Deb proposed the NSGA-II algorithm (Deb et al., 2002; Haupt and Werner, 2007; Sahoo et al., 2009; Sivanandam and Deepa, 2008; Subramanian et al., 2009) which essentially differs from non-dominated sorting genetic algorithm (NSGA) implementation in a number of ways as follows: NSGA-II uses an elite-preserving mechanism; uses a fast non-dominated sorting procedure; does not require any tunable parameter to make the algorithm independent of the user experience. Initially, a random parent population is created. The population is sorted based on the non-domination. A special book-keeping procedure is used in order to reduce the computational complexity. Each solution is assigned a fitness equal to its non-dominated level (1 is the best level). Thus, minimisation of fitness is assumed. Tournament selection, recombination, and mutation operators are used to create a child population, thereafter; an algorithm was developed and followed in every generation as described in Sivanandam and Deepa (2008).

5 Results and discussion

The multiobjective optimisation problems, equations (7) and (10) were solved using a computer code for NSGA-II (Deb et al., 2002; Haupt and Werner, 2007; Sahoo et al., 2009; Sivanandam and Deepa, 2008; Subramanian et al., 2009).The computational parameters used in this study are given in Table 2. The bounds for the optimisation parameters, d, β, a, U, are given in Table 3. The NSGA-II programme is used to find the optimal values of the decision parameters to minimise the above two functions and for each solution of the NSGA-II, the programme calls the SGA-CSM programme within a local loop. The bounds used on the four decision variables have been chosen to encompass the values corresponding to laboratory-type experimental electrostatic separators data published in literature (Das et al., 2007; Iuga et al., 2011; Medles et al., 2007; Vlad et al., 2000) with the aim that the present study will suggest a more optimal configuration and operating parameters. Compactness of electrostatic separators is a significant design requirement; accordingly a rather narrow range of variation is attempted for all parameters. An upper bound for U =40 kV, coupled with a lower bound for d = 0.07 m, is assumed to allow a safe margin from a sparkover condition. The upper bound for d is selected, randomly, to be 0.1 m. Effect of variation of the high voltage electrode length has not been, earlier, considered as to its influence on the field distribution on the ground plate. A relatively small range for the electrode length, a = 0.07–0.09 m, is, here, taken to circumvent a value of 0.075 m commonly used in earlier literature (Vlad et al., 2000). A common value for the inclination of the ground plate α = 60o is usually used in laboratory-type separators, and is also used here to compute the detachment field equation (4) for the different species. The maximum slope allowed for the high voltage electrode, in laboratory-type separators, is β = 30° which is taken here to represent the upper bound for its variation; while the lower bound β = 0° represents the case of the electrode being parallel to the ground plate.

Table 2 Computational parameters for real code NSGA-II

NSGA-II (parameters)	*Parameter values (type)*
Population size (P_s)	20
Number of generations (N_g)	50
Crossover probability (P_c)	0.9
Mutation probability (P_m)	0.2
Cross over index (η_c)	50, (simulated binary crossover)
Mutation index (η_m)	50, (polynomial mutation)

Table 3 Bounds of optimisation parameters

Optimisation parameters	*Lower bound*	*Upper bound*
d	0.07	0.10 (m)
β	0	30 (deg)
a	0.07	0.09 (m)
U	10	40 (kv)

The method is applied for particles of different sizes and different specific masses. The Pareto sets for the case of two particles with $r = 50 \times 10^{-3}$ mm, $\gamma_1 = 3{,}000$ kg/m^3 and $\gamma_2 = 9{,}000$ kg/m^3 are shown in Figure 8. This diagram shows some scatter in the distributions of the feasible solutions in the early generations. However, the Pareto set starts to emerge from about the fifth generation and to be essentially complete at $N_g = 50$. The results of Figure 8 reveal that, as is common for problems involving multiple objective functions, there may exist several acceptable optimal points or non-dominated solutions and any one of them could, equally, be selected for design or operation. The choice of anyone solution from among the Pareto set of points requires additional knowledge and experience and may be intuitive. However, the Pareto set is of prime importance in narrowing down the choices to be considered. Close review of earlier numerical, experimental and physical analysis of the plate-type separator process leads to the belief that the satisfaction of the objective function F_1 would be more significant than that of F_2. Consequently, solutions with the lowest value for F_1 were selected as the optimal solutions of the problem. These results are reported in Tables 4 to 6 and field distributions are plotted in Figures 9(a) to 9(c).

Table 4 reports the optimal values of the optimisation parameters and the corresponding values of X_{d1}, X_{d2}, $X_{d2} - X_{d1}$, for r between 50×10^{-3} and 500×10^{-3} mm and for $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3. Similar results are reported in Table 5 for $\gamma_1 = 3{,}000$, $\gamma_2 = 6{,}000$ kg/m^3 and in Table 6 for $\gamma_1 = 6{,}000$, and $\gamma_2 = 9{,}000$ kg/m^3. These results demonstrate the effectiveness and success of application of NSGA-II for solving the complex multiobjective optimisation of the plate-type electrostatic separator.

The data presented in Tables 4 to 6 indicate that the separation of a mixture containing particles of same radius ranging between 50×10^{-3} and 500×10^{-3} mm and mass density between $\gamma_1 = 3{,}000$ kg/m^3 and $\gamma_2 = 9{,}000$ kg/m^3 is feasible; i.e., $X_{d2} > X_{d1}$, in all the cases considered. Larger separation distances ($X_{d2} - X_{d1}$) are observed for binary mixtures with higher γ_2 / γ_1 ratio (e.g., $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3) than those for lower ones (e.g., $\gamma_1 = 3{,}000$ and $\gamma_2 = 6{,}000$ kg/m^3 or $\gamma_1 = 6{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3) rendering the separation more feasible in such cases. This may be attributed to the proposed objective function F_2 which seeks to maximise the separation distance ($X_{d2} - X_{d1}$) hence the selectivity and efficiency of the separator. Some inconsistency in the results for ($X_{d2} - X_{d1}$) was observed; e.g., the case $r = 400 \times 10^{-3}$ mm in Table 4, which may be related to the selection of optimal solutions with lowest value of F_1 on the expense of F_2. It should be noted that larger separation distances are usually preferred due to their subsequent impact on the particles' trajectories and consequently the collection efficiency. Finer grinding is always necessary for processing heavy materials in Plate-type separators (Vlad et al., 2000). In this optimisation study, the algorithm, with the proposed bounds and procedures, has the ability to detach particles of radii between 50×10^{-3} and 500×10^{-3} mm for the three different values of mass density considered.

It is interesting to observe from Figure 9 and the results of Tables 4 to 6 that lighter particles; i.e., lower γ, always detach within the first half of the plate ($X_{d1} \leq L/2$) for practically all the cases considered. However, for higher γ_2 /

γ_1, heavier particles will practically detach on the second half of the plate while for moderate and smaller γ_2 / γ_1, only larger particles($r \geq 200 \times 10^{-3}$) will do so. This may be attributed to the variation in the slope of the rising part of the field profile as well as its magnitude (Figure 9), which is controlled by the values of the decision variables and the enforcement of the objective functions F_1 and F_2. It is noted that the optimal values of the decision variables (d and a) lie, practically, at their bounds (a at its upper bound and d at its lower bound). The selection of the bounds of these variables as the optimal values of the decision variables by the algorithm can be explained physically. For example, the need to achieve an electric field value of a magnitude equal or larger than both E_{d1} and E_{d2}, a decrease in the value of d seems to be more influential than increasing the potential U. Thus, to increase the field strength the method selects d at its lower bound while the potential U remains reasonably well within its bounds. In addition, the method sets the upper bound of a as the optimum value in order to exert more modulation in the field profile reflecting the lesser effect of the inclination angle β which lies reasonably well within its bounds. It is also of interest to observe the behaviour of the method for the case of the largest particle considered ($r = 500 \times 10^{-3}$ mm). In this case, the method sets all the decision variables at, practically, either their lowest or highest bounds.

Figure 8 The Pareto set for $r = 50 \times 10^{-3}$ mm, $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3 (see online version for colours)

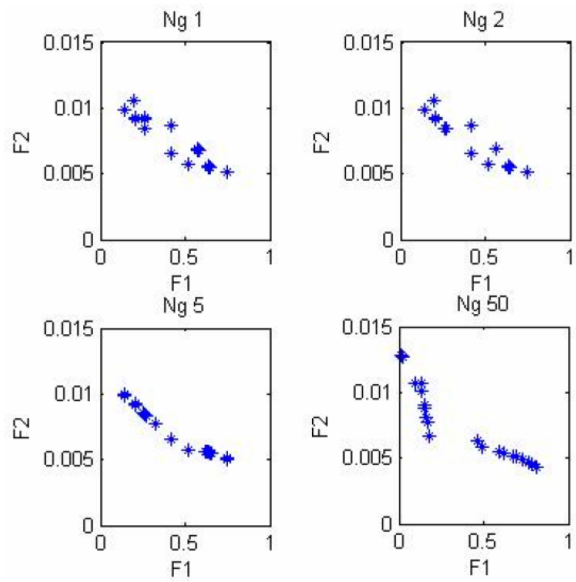


Table 4 Optimal parameters for $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3 (Best F_1)

$r \times 10^{-3}$	d	U	β	a	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.070	14.976	26.430	0.089	119.917	197.690	77.773
100	0.070	23.639	19.477	0.090	97.003	188.565	91.562
200	0.084	37.924	26.505	0.090	103.973	192.198	88.225
300	0.071	37.110	26.129	0.090	118.480	197.553	79.074
400	0.070	37.151	28.290	0.090	138.943	210.701	71.757
500	0.070	40.000	30.000	0.090	144.768	211.483	66.715

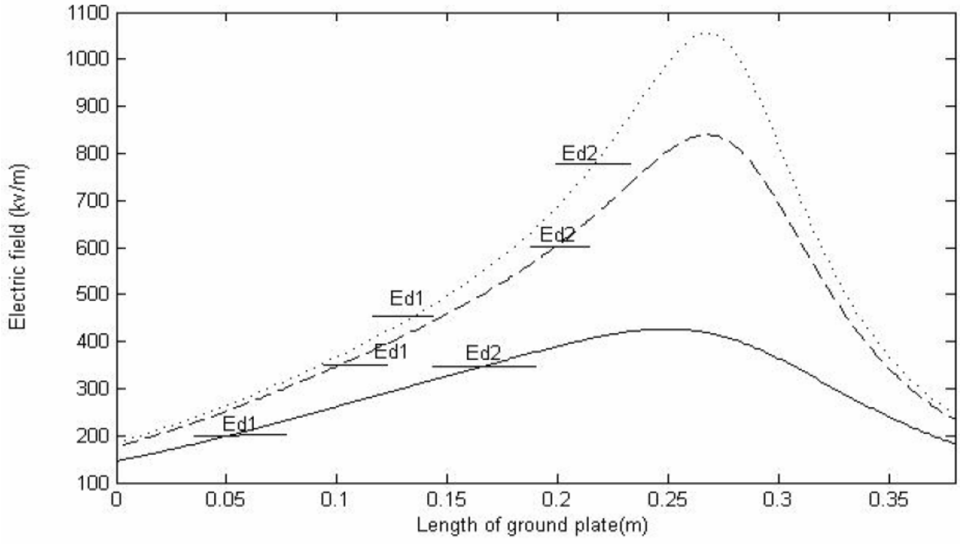
Table 5 Optimal parameters $\gamma_1 = 3{,}000$ and $\gamma_2 = 6{,}000$ kg/m^3 (Best F_1)

$r \times 10^{-3}$	d	U	β	a	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.083	17.663	15.903	0.090	101.335	170.344	69.008
100	0.079	24.726	17.365	0.081	104.408	163.651	59.243
200	0.098	39.335	25.622	0.090	116.214	182.485	66.270
300	0.076	38.621	14.865	0.090	107.267	178.413	71.146
400	0.070	38.868	14.345	0.090	117.107	192.705	75.599
500	0.070	40.000	24.210	0.090	142.021	195.853	53.831

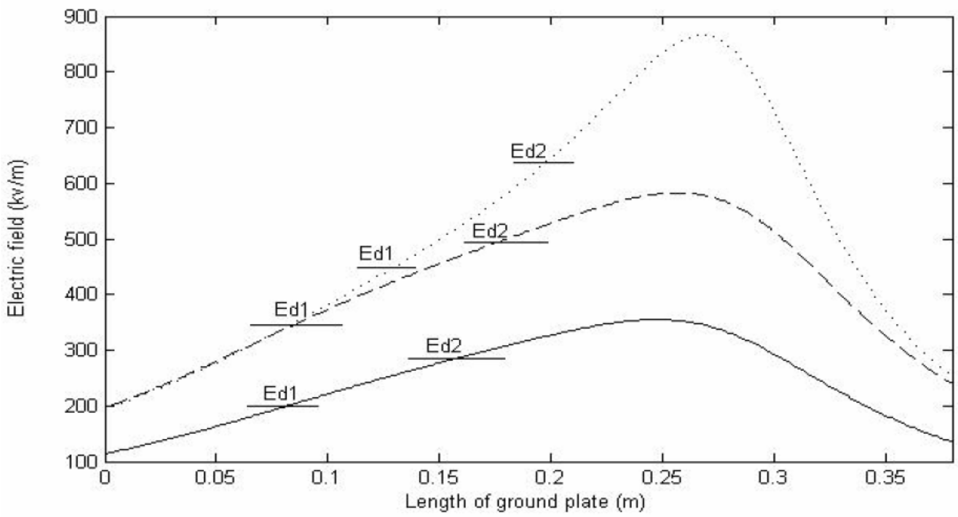
Table 6 Optimal parameters for $\gamma_1 = 6{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3 (Best F_1)

$r \times 10^{-3}$	d	U	β	a	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.087	26.600	17.044	0.071	111.671	144.319	32.648
100	0.099	39.030	13.765	0.090	107.947	154.813	46.866
200	0.070	38.084	7.273	0.090	107.445	151.376	43.931
300	0.071	37.407	26.367	0.089	167.464	196.187	28.723
400	0.070	37.378	26.178	0.090	187.259	214.443	27.184
500	0.070	39.981	26.970	0.090	192.432	218.214	25.781

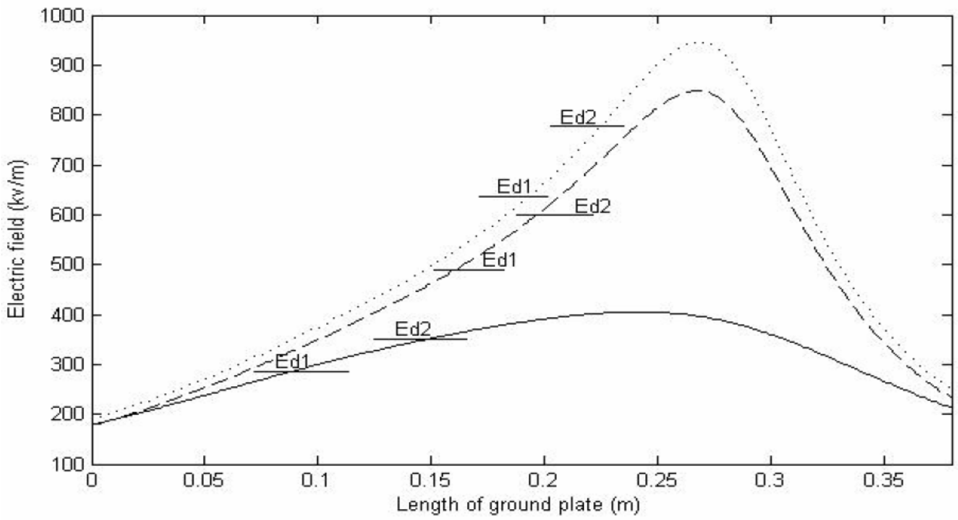
Figure 9 Electric field on the ground plate for different particle radii, (a) $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3 (b) $\gamma_1 = 3{,}000$ and $\gamma_2 = 6{,}000$ kg/m^3 (c) $\gamma_1 = 6{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3



(a)



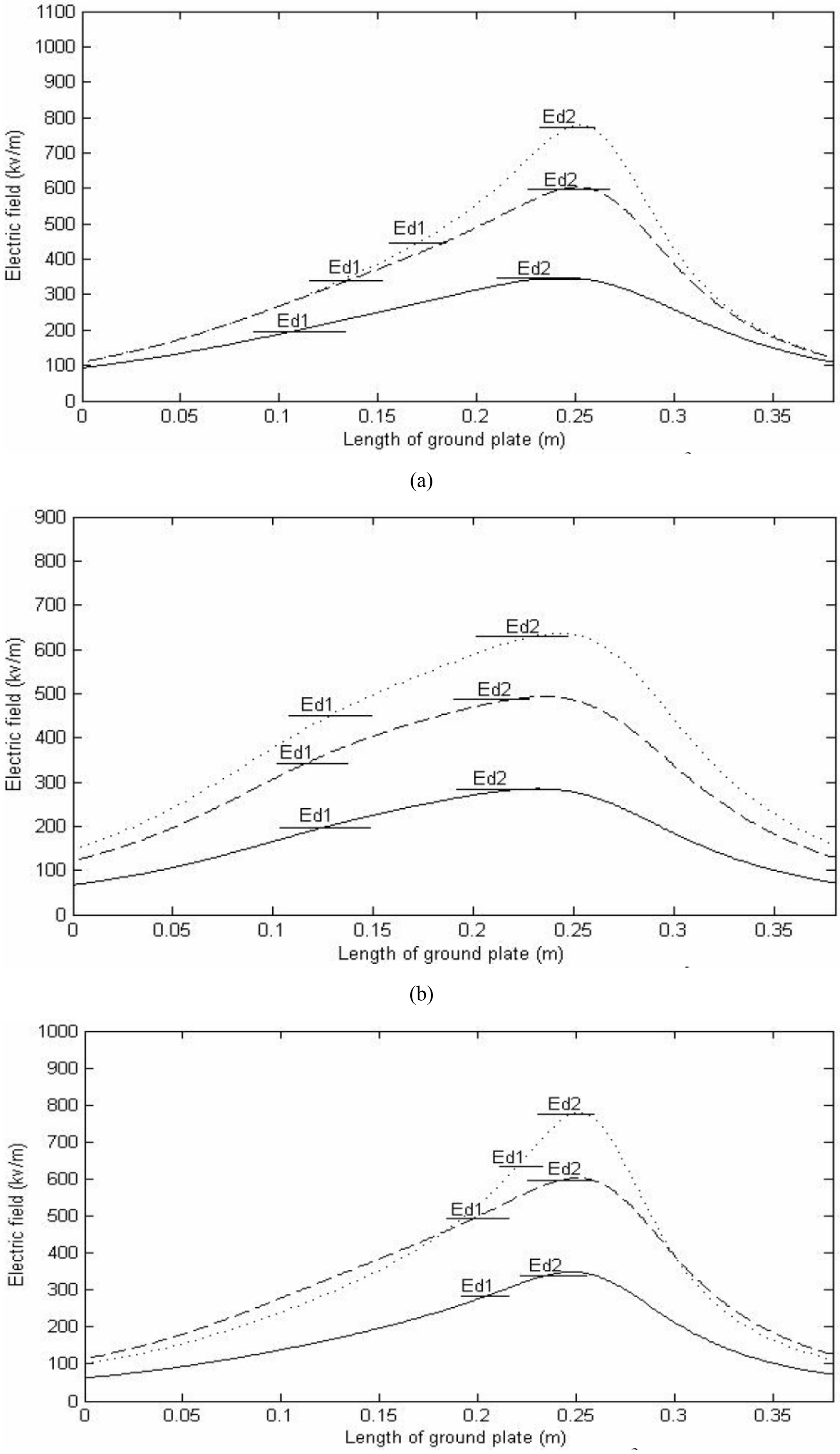
(b)



(c)

Notes: ___ r = 100 × 10^{-3} mm, - - - r = 300 × 10^{-3} mm, ... r = 500 × 10^{-3} mm

Figure 10 Electric field on the ground plate for different particle radii (Best F_2), (a) $\gamma_1 = 3{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3 (b) $\gamma_1 = 3{,}000$ and $\gamma_2 = 6{,}000$ kg/m^3 (c) $\gamma_1 = 6{,}000$ and $\gamma_2 = 9{,}000$ kg/m^3



(a) (b) (c)

Notes: ___ r = 100 × 10^{-3} mm, - - - r = 300 × 10^{-3} mm, ... r = 500 × 10^{-3} mm

It is recalled here that for problems involving multiple objective functions, a set of several equally desirable optimal points (Pareto sets) exists; and anyone of them could be selected for design or operation as described above. These Pareto sets are of significant importance in narrowing down the choices to be considered by designers. In order to further demonstrate this feature of multiobjective optimisation, an alternate solution, out of the Pareto set, is selected and examined. The solution with the best value of F_2 is selected which means that all the other Pareto solutions would be between this solution and the solution of the best F_1 analysed above. The results for the solution with best F_2 are reported in Tables 7 to 9 and field distributions are plotted in Figures 10(a) to 10(c). The data presented in Tables 7 to 9 indicate again, similar to the case with best F_1, that the separation of a mixture for all the cases considered is still feasible; i.e., $X_{d2} > X_{d1}$. Also, the present values of $X_{d2} - X_{d1}$ are, generally, larger than the corresponding values of $X_{d2} - X_{d1}$ of the first case. This would result in larger separation distances; hence more distinct trajectories and better collection efficiency, especially for mixtures of lower γ_2 / γ_1. However, a possible drawback of using the present solution is that the detachment field E_{d2} is quite close to the maximum field on the ground plate as shown in Figures 10(a) to 10(c). For example E_{d2} = 778.8 kV/m for r = 500 mm and γ_2 = 9,000 kg/m^3, while E_{max} = 779.14 kV/m in this case [Figure 10(a)]. A slight variation in the decision variables such as lower voltage and/or deviation in the electrode angle β may lead to a lower E_{max} with the result that the particles not being detachment from the plate. The previous analysis and examination of the solutions for F_1 and F_2 demonstrate clearly the effectiveness of the presented approach and its benefits for the study, design and development of electrostatic separators. The obtained Pareto solutions assisted in narrowing down the choices faced by decision maker for establishing optimal performance of the plate type electrostatic separator.

Table 7 Optimal parameters for γ_1 = 3,000 and γ_2 = 9,000 kg/m^3 (Best F_2)

$r \times 10^{-3}$	*d*	*U*	*β*	*a*	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.081	13.453	28.232	0.090	152.379	246.083	93.703
100	0.100	29.057	25.235	0.090	110.511	255.101	144.590
200	0.096	39.254	24.779	0.090	113.385	241.185	127.800
300	0.071	31.503	20.847	0.090	138.617	249.221	110.604
400	0.072	34.351	23.340	0.090	150.773	267.101	116.328
500	0.070	33.035	26.921	0.090	171.068	251.137	80.070

Table 8 Optimal parameters γ_1 = 3,000 and γ_2 = 6,000 kg/m^3 (Best F_2)

$r \times 10^{-3}$	*d*	*U*	*β*	*a*	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.078	11.070	26.448	0.088	179.577	246.257	66.681
100	0.077	20.340	14.218	0.080	128.512	228.691	100.179
200	0.091	32.568	21.015	0.084	138.074	233.953	95.880
300	0.074	33.973	10.833	0.086	119.301	231.595	112.295
400	0.070	32.207	16.763	0.090	155.201	245.704	90.503
500	0.070	39.788	11.775	0.090	127.871	241.520	113.650

Table 9 Optimal parameters for γ_1 = 6,000 and γ_2 = 9,000 kg/m^3 (Best F_2)

$r \times 10^{-3}$	*d*	*U*	*β*	*a*	X_{d1}	X_{d2}	$X_{d2} - X_{d1}$
50	0.078	15.643	20.310	0.086	183.830	257.090	73.260
100	0.082	19.306	29.603	0.089	204.360	244.455	40.096
200	0.084	31.559	24.350	0.090	193.020	245.201	52.182
300	0.071	32.231	19.558	0.090	197.396	265.090	67.695
400	0.070	32.622	23.853	0.090	210.539	270.100	59.561
500	0.070	30.371	29.778	0.089	220.092	249.535	29.442

The present study illustrates the applicability of multiobjective optimisation techniques to, effectively, model plate type separators. Further research is needed to refine the present model by including more decision variables, simulating the optimal separation trajectories after detachment from the plate electrode, the collection efficiency as well as including other effects such as particle interactions.

6 Conclusions

Multiobjective optimisation of the plate type electrostatic separator was carried out using NSGA-II technique. An illustrative configuration to enhance the selectivity; hence the efficiency, of the separator through compromisation of four decision variables is presented. The developed GA-CSM code proved to be an effective tool for the analysis of the electric field in this type of installations. Procedures were devised, through enforcement of two proposed objective functions, aiming to enhance the separator selectivity and performance. Analysis of the four decision variables included in the optimisation procedures showed that the gap distance d may be more influential than the voltage U for field enhancement while the high voltage electrode length is more important than the inclination angle β in modulating the field profile. The Pareto solutions obtained in this work assisted in narrowing down the choices faced by decision maker for optimal performance of the plate type electrostatic separator. The optimum operating parameters of the plate type electrostatic separator would have to be specifically established for each granular material, taking into account the characteristic features of each constituent.

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